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Axioms for the Weighted Linear Model

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UNIVERSITY OF COLORADO

INSTITUTE OF BEHAVIORAL SCIENCE

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An axiomatization is provided for the weighted linear model for the case defined by three conditions: (a) two and only two attributes; (b) sparse, finite alternatives and attribute levels; and (c) constant attribute-level scale values across all orderings. Theorems outlining necessary conditions, sufficient conditions, and uniqueness for this case are also presented. The axiomatization is compared to a similar effort by Luce (1980). It is hoped that the axiomatization can help in identifying and exploring areas of common ground and disagreement among various approaches to judgment and decision

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Abstract, continued

making. Implications of the results in three areas are discussed as examples: the identifiability of weights, the uniqueness of estimates of weights and scale values, and the effect of the range of attribute levels on weight.

The weighted linear model is used to represent an individual's judgments and decisions in many diverse approaches to judgment and decision making (e.g., see Hammond, McClelland, & Mumpower, 1980). This convergence on a common model should provide an important source of agreement among different approaches. Instead, the weighted linear model has often been the battleground for arguments between proponents of the various approaches. For example, the model has been operationalized in a variety of ways, each claimed to be superior by its proponents. There is an especially wide variety of methods for measuring the weight parameters in the model. There is also much disagreement about the interpretation of weights. Some approaches treat weights as mere scaling constants with no psychological content (e.g., Keeney & Raiffa, 1976) or as "empirically empty parameters" (Schönemann, Cafferty, & Rotton, 1973). On the other hand, proponents of other approaches claim that weight parameters are empirically identifiable and are psychologically meaningful in that they reflect relative importance or salience (e.g., Anderson, 1970, 1973; Hammond, Stewart, Brehmer, & Steinmann, 1975). Clearly, the potential agreement suggested by the ubiquitous use of the weighted linear model has not been achieved.

The purpose of this paper is to provide an axiomatization for the weighted linear model. The axiomatization provides necessary and sufficient conditions for a particular case of the weighted linear model, thereby furthering fundamental understanding of the model itself. The axiomatization should aid in the effort to identify common ground and sources of disagreement among various approaches to judgment and decision making.

The organization of this paper is as follows: (1) definition of the weighted linear model; (2) a consideration of five psychologically important characteristics of the concept of weight, a concept central to the weighted linear model; (3) a description of the conditions under which the axiomatization is applicable; (4) an illustrative example; (5) the axioms; (6) the theorems; (7) scaling procedures; and (8) discussion and implications.

The Weighted Linear Model

Because the weighted linear model appears in so many guises under diverse names in the various approaches, we will first establish a common formal definition of the model. The representation of the weighted linear model which we will use is as follows: there are several attributes (or cues or factors or dimensions, depending upon the approach) $X_1, X_2, \ldots, X_k$, and each is associated with a numerical weight w_i and a real-valued function f_i such that the numerical value assigned to the alternative (or stimulus complex or profile) represented by $(x_1, x_2, \ldots, x_k)$ is given by

$$F(x_1, x_2, \ldots, x_k) = \sum_{i=1}^{k} w_i f_i(x_i)/(\Sigma w_i).$$

Anderson (1970, 1974) refers to this model as the "averaging model".

Because we will focus on the two-attribute case, it is convenient to introduce a simpler notation for that case. Let one attribute be represented by the set $A = \{a, b, c, \ldots\}$ where the lowercase letters represent specific attribute levels, and the other attribute by $P = \{p, q, r, \ldots\}$. The normalized weighted linear model, then, is given by

$$F(a, p) = wf(a) + (1-w)g(p),$$

where w is the numerical weight associated with the first attribute and f and g are real-valued functions representing evaluations of the separate attributes.

Characterizing the Concept of Weight

When an individual must choose among alternatives varying on several attributes, choice of the alternative with the "best" value on all attributes is often precluded. The individual must then compromise by assigning more weight to some attributes than to others. The concept of weight reflects this process of compromise or trade-off.

The definition of the weighted linear model presented above captures five important properties of weight, properties which seem essential to any representation of the psychological concept of weight.

1. Weights are associated with attributes (or cues or factors) rather than with specific attribute levels. That is, it is the attribute that is "weighted" rather than any particular instance or level of that attribute.

2. Weights are independent of the attribute evaluation functions (i.e., f and g) in the sense that the choice of specific weights does not constrain the possible functions for f and g, and the choice of

specific evalution functions does not constrain the possible weights. It is thus logically possible for any set of weights to be combined in the model with any evaluation functions. If this were not so the "weight" could be absorbed into the evaluation function itself, and the model would reduce to a simple adding model.

3. Weights interact with the evaluations of the attribute levels to determine the overall evaluations. If the weights did not interact with the evaluation functions we would have the following additive model:

$$F(a, p) = w_1 + f(a) + w_2 + g(p) = f(a) + g(p) + k.$$

Clearly, the "weights" in such a model reduce to a constant term added to the output of the evaluation functions. The constant cannot affect the relative evaluation of the alternatives, so the "weights" would have no real impact. The interaction between weights and evaluation functions in the weighted linear model is represented by multiplication. The nature of the interaction is as follows: for a specific attribute level to have a significant impact on the overall evaluation, the evaluation of that attribute level must have a reasonable magnitude <u>and</u> the weight associated with that attribute must be appreciable. If either the magnitude of the evaluation is trivial or the weight is close to zero then that attribute level would have virtually no impact on the overall evaluation.

4. The weights must sum to some constant K (in the above definition, K = 1). Without this constraint the weighted linear model again reduces to a simple adding model. That is, there is no

compromise unless the weights are constrained.

5. In order for weights to be interesting parameters in the analysis of judgment and decision making, they must vary in two different aspects. First, weights must vary across attributes; otherwise, the constant weights could be omitted from the model without loss, reducing it to a simple adding model. Second, the weights must vary for the same person across situations and times and/or vary across people. If such variation does not exist then there is little point in estimating weights in either research or applications involving the weighted linear model.

Conditions of the Axiomatization

The axiomatization presented below is applicable to the following special case of the weighted linear model: (1) two and only two attributes (vs. a variable number), (2) finite or limited and sparse attribute levels (vs. infinite levels), and (3) common attribute-level scale values in all evaluations of the alternatives. The importance of each of these conditions will be discussed briefly.

Constant vs. variable number of attributes. Two methods have been used to examine variance in relative weights in judgment and decision tasks. One is to vary "set-size"; that is, to vary the number of attributes that are used to describe the alternative (e.g., Norman, 1976a,b). The logic of this method is that the weight on a given attribute must increase when the number of attributes is decreased, say, from k to k-1, because in either case the sum of the weights must be the same; if fewer weights must add up to the same constant, then any individual weight must increase. The second method keeps the number of attributes constant and examines either differences across people or differences in judgments made by the same person under different conditions or assumptions. For example, an individual making evaluative judgments of automobiles might place different weights on attributes such as comfort, safety, and fuel efficiency when selecting a car for personal use than when making a recommendation for a car to be used in a business fleet. The axiomatization provided here specifies a constant number of attributes and therefore cannot be used with set-size research. Such research has been limited primarily to information integration theory (see Anderson, 1974).

Finite vs. infinite case. An axiomatization could be constructed for the infinite case--assuming dense alternatives and attribute levels--or for the finite case, assuming limited and sparse alternatives. There are advantages and disadvantages to both approaches. Axioms for the infinite case often lead to a better understanding of the representation and of its relationship with other measurement structures. However, axioms for the infinite case are often not empirically testable in the finite designs actually used. Although the finite case usually leads to a set of empirically-testable axioms, there is often no finite set of axioms sufficient for all cases (e.g., Scott & Suppes, 1958); that is, the number of axioms depends upon the number of attributes and the number of levels on each attribute. We have chosen the finite case for our analysis because we want empirically-testable axioms in order to determine whether it is possible both theoretically and practically to measure weights.

Common attribute-level scale values. As noted above, any evaluation of an alternative is the result of an interaction of the evaluations (scale values) of the attribute levels with the weights. This implies that the concept of weight--and the task of providing an axiomatization for the weighted linear model--is meaningless unless common attribute-level scale values are assumed to hold across different people and/or across the evaluations made by the same person under different conditions. This is a commonly-made assumption in judgment and decision-making research. For example, Anderson (1973) notes that a method he has frequently used to estimate weights requires "an assumption that the scale values, or at least their differences, are constant . . . (p. 91)." The import of this assumption is discussed in more detail later.

Luce's axiomatization. Luce (1980) provides an axiomatization for the weighted linear model in the infinite case with variable numbers of attributes. Later in this paper we compare our results to his in order to determine whether the choice of one case or another has any important consequences.

An Example

An example will provide a basis for understanding how the axioms work. The focus of this example is the evaluation of the desirability of nine alternative prizes (commodity bundles) for a competition. The nine alternatives (listed in Table 1) constitute the set of all possible combinations of 0, 2, or 5 phonograph records with 0, 1, or 5 books. The judge's task is to rank order the nine alternatives. Given two such rank orders, is it possible to determine which represents the greater weight on, say, the record attribute?

If the judge places almost all the weight on records, then the following lexicographic ordering would be obtained: ABCDEFGHI. If instead all the weight is placed on books, the ordering would be ADGBEHCFI. Other orderings would represent intermediate weightings between these two extremes. This suggests using the number of pair reversals between a given ordering and one of the lexicographic orderings as an ordinal measure of distance. Such a measure could then be used to compare the relative weighting implied by two orderings, assuming that the evaluation functions used to generate each ordering were the same. For example, consider the following two orderings:

S1: A B C D E G H F I

S2: A B D G E C H F I

These orderings might be from two different people or from the same person at two different times. The ordering S1 is two pair reversals (F-H and F-G) from the lexicographic ordering which places almost all the weight on records, while S2 is six pair reversals away. Thus, S1 appears to place greater weight on records than does S2.

The goal of the axiomatization is to specify how and when a common representation can be derived for a set of orderings (S1, S2, that would occur as increasing weight is placed upon the book attribute.

Parallel to unfolding theory. The problem of developing an axiomatization for the weighted linear model is essentially equivalent to answering the following question: What are the necessary and sufficient conditions on a set of orderings (such as those in Table 2) that would allow construction of a graph such as Figure 1? Readers familiar with unfolding theory (Coombs, 1964) will note the strong similarity between our ordering of pair reversals and the ordering of midpoints in unfolding theory. The problem in unfolding theory is to describe the conditions necessary for a set of individual orderings ("I-scales") to be "unfolded" so that they can all be represented in terms of a common or joint scale ("J-scale") of stimuli; each individual ordering is represented by an interval (a region between two adjacent midpoints) on the joint scale. Similarly, the problem here is to describe the conditions necessary for a set of individual orderings to be "unfolded" into a common or joint space (e.g., Figure 1) involving two stimulus scales, one for each attribute; an individual ordering is represented by an interval (a region between two adjacent intersections) on a line between the two scales. Thus, we should expect to find many similarities between the two problems.

For a set of I-scales to be unfolded, they must all end with one of only two stimuli, the stimuli which define the ends of the J-scale. A similar constraint is obvious here. All orderings must begin with the same alternative and all orderings must end with the same alternative. Since it is assumed that the evaluation functions are the same for all orderings, it is impossible for a change in weighting to affect the relative position of either the first or last alternatives. This is apparent in Figure 1. Thus, the first and last choices for all orderings must be identical.

Another important diagnostic for unfolding is that two I-scales from a common quantitative J-scale cannot imply different midpoint orderings. A similar condition applies to the weighted linear model. Two orderings cannot imply different orderings of the intersections if a common representation exists. For example, consider the following two orderings:

S3: A B C D E G F H I

S4: A D B E C F G H I

S3 has one pair reversal and thus must be one intersection to the right of the lexicographic ordering ABCDEFGHI; that is, S3 implies that FG is the first intersection. S4 has three pair reversals, implying that it must be further to the right than is S3 and thus to the right of the FG intersection. However, the absence of a reversal between F and G in S4 implies that S4 is to the left of the FG intersection. This obviously is an impossible set of conditions; S4 cannot be on both the left and right sides of the FG intersection. S3 and S4 thus imply different orderings of the intersections. They are incompatible in terms of the weighted linear model and cannot both be represented on one graph like Figure 1.

The above example suggests that we want a set of axioms that will ensure that we can construct a transitive ordering of the intersections. The example also suggests a very simple but powerful empirical test of whether a representation consistent with the weighted linear model can be constructed: If ordering X has more pair reversals (relative to a lexicographic ordering) than ordering Y, then all the pair reversals contained in Y must also be contained in X. In other words, if ordering X has x pair reversals and ordering Y has y, then there must be exactly $|x - y|$ pair reversals between orderings X and Y. The following axiomatization formalizes and extends this simple empirical test.

<u>The Axioms</u>

Four axioms are required for the two-attribute case. We begin with the following primitives:

$A = \{a, b, c, \ldots\}$ and $P = \{p, q, r, \ldots\}$ and

$I = \{\succsim_i, \succsim_j, \succsim_k, \ldots\}$

A and P are sets representing the attributes and a, b, c, etc. are specific attribute levels. I is a set of binary relations each of which is defined on A x P. Where it will not cause confusion, we will simply use i, j, k, etc. to refer to these binary relations. (These binary relations, which we hope will be transitive orderings of the alternatives, may be generated by the same person at different times or by different people.) The following definition establishes the representation we are seeking.

<u>Definition</u>: <<A x P, I>> is a <u>weighted linear structure for two attributes</u> if and only if there exist real-valued functions f and g

defined on A and P, respectively, and real parameters w_i in the interval [0, 1] for all i in I such that

i) $(a, p) \succsim_i (b, q)$ iff $F(a, p) \geq F(b, q)$, and

ii) $F(a, p) = w_i f(a) + (1-w_i)g(p)$.

That is, a weighted linear structure exists if and only if all the orderings can be represented by a common set of evaluation functions with only the weight parameter varying across orderings.

<u>Axiom 1</u>: <u>Within-ordering additivity</u>

$\succsim_i$ satisifes the axioms of additive conjoint measurement for all i in I.

For any single ordering we can define $f'(a) = w_i f(a)$ and $g'(p) = (1-w_i)g(p)$ so that (ii) in the above definition reduces to the simple additive model. Thus, any single ordering must satisfy the axioms of additive conjoint measurement (Krantz, Luce, Suppes, & Tversky, 1971).

<u>Axiom 2</u>: <u>Cross-ordering monotonicity</u>

For all a, b in A; p, q in P; and i, j in I:

i) $(a, p) \succsim_i (b, p)$ implies $(a, p) \succsim_j (b, p)$

ii) $(a, p) \succsim_i (a, q)$ implies $(a, p) \succsim_j (a, q)$.

If all the orderings are to be represented by the same evaluation functions then the rankings of rows and columns in the A x P matrix must be the same in each ordering because multiplying the evaluation functions by positive weights cannot possibly reverse the order. Formally, $(a, p) \succsim_i (b, p)$ implies

$$w_i f(a) + (1-w_i)g(p) \geq w_i f(b) + (1-w_i)g(p), \text{ or } f(a) \geq f(b).$$

Because this last inequality does not depend upon the ordering i, it must be true for all orderings, hence Axiom 2(i). Axiom 2(ii) is equally trivial.

<u>Axiom 3: Cross-ordering cancellation ("Weighting axiom")</u>

i) If there exists an ordering such that

$(a, q) >_i (b, p)$ and $(c, p) \succsim_i (b, q)$ then

$(b, s) >_j (c, r)$ implies $(a, s) >_j (b, r)$

for all a, b, c in A; all p, q, r, s in P; and all i, j in I.

ii) If there exists an ordering such that

$(a, q) >_i (b, p)$ and $(b, q) \succsim_i (a, r)$ then

$(c, r) >_j (d, q)$ implies $(c, q) >_j (d, p)$

for all a, b, c, d in A; all p, q, r in P; and all i, j in I.

[Note: $(a, q) > (b, p)$ means not $(b, p) \succsim (a, q)$.]

Axioms 1 and 2 ensure that it makes sense to consider an ordering of intersections. Axiom 3 does most of the work of ensuring that the ordering is transitive. Because of its importance and because it implies the simple empirical test described above, it seems reasonable to label Axiom 3 the "weighting axiom".

Axiom 3 can be explained in the context of the record-book example. Let r = p and s = q. Then Axiom 3(i) simply states that if F = (b, p) and H = (a, q) have been reversed [i.e., $(a, q) \succsim (b, p)$] but C = (c, p) and E = (b, q) have not, then F and H also must be reversed in any other ordering in which C and E have been reversed. To state this differently, the first line of Axiom 3(i) implies that the CE intersection is to the right of the FH intersection. The second line of Axiom 3(i) then requires that any ordering to the right of the CE intersection also must be to the right of the FH intersection. Thus, Axiom 3 precisely describes the empirical test described above.

Axiom 3 also imposes additional constraints on the ordering of the intersections. Continue the example and let $s = r$ and $r = q$. Axiom 3(i) then requires that in any ordering in which B = (c, q) and D = (b, r) have been reversed, E = (b, q) and G = (a, r) also must be reversed. That is, if the FH intersection is before (to the left of) the CE intersection, then the EG intersection must come before the BD intersection--crossing the BD intersection requires crossing the EG intersection as well. Further, let $s = r$ and $r = q$; then Axiom 3(i) requires that the FG intersection must come before the CD intersection. These requirements reduce the number of possible intersection orderings consistent with the weighted linear model and given orders of the attribute levels from 9! to 48, a finding discussed later. Not all of these implications of Axiom 3(i) are obvious from the example. The following proof of the necessity of Axiom 3(i) justifies these results as well as the name "cancellation".

<u>Proof of Necessity of Axiom 3(i)</u>:

If $(a, q) >_i (b, p)$ and $(c, p) \succsim_i (b, q)$ then there exists w such that

$$wf(a) + (1-w)g(q) > wf(b) + (1-w)g(p) \text{ and}$$

$$wf(c) + (1-w)g(p) \geq wf(b) + (1-w)g(q).$$

Adding these two inequalities yields

$$w[f(a) + f(c)] + (1-w)[g(p) + g(q)] > w[2f(b)] + (1-w)[g(p) + g(q)].$$

Cancelling the common term $(1-w)[g(p) + g(q)]$ gives

$$w[f(a) + f(c)] > w[2f(b)].$$

Assume that $w > 0$. (If $w = 0$, then the first two inequalities reduce to $g(q) \geq g(p)$ and $g(p) \geq g(q)$ which imply $g(p) = g(q)$, leading to a trivial realization of the weighted linear model.) Then, dividing by w, the previous inequality is equivalent to

$$f(a) + f(c) > 2f(b).$$

Now if $(b, s), >_j (c, r)$ and if, contrary to Axiom 3(i), $(b, r) \not\succsim_j (a, s)$, then there exists z such that

$$zf(b) + (1-z)g(s) > zf(c) + (1-z)g(r) \text{ and}$$

$$zf(b) + (1-z)g(r) \geq zf(a) + (1-z)g(s).$$

Performing the same cancellation as above yields

$$2f(b) > f(a) + f(c).$$

However, this last result contradicts the implication derived above that $f(a) + f(c) > 2f(b)$. Thus assuming that $(b, r) \not\succsim_j (a, s)$ leads to a contradiction; hence, Axiom 3(i). The proof for Axiom 3(ii) is similar.

Figure 2 shows a graphic representation of Axiom 3. The single arrows represent the conditions of the axiom; the head of each arrow indicates the higher-ranked of the two cells connected by the arrow. The double arrow represents the implication.

Even though Axiom 3 ensures that the ordering of the intersections will be transitive, it is not sufficient to enable the construction of a weighted linear representation. The proof that Axioms 1, 2, and 3 are not sufficient is rather tedious. However, we present it below because it leads directly to Axiom 4 and because it also suggests how the actual scaling of the parameters might be accomplished.

Proof that Axioms 1, 2, and 3 are not sufficient:

Assume, consistent with Axiom 2, that

$(c, p) \succsim (b, p) \succsim (a, p)$ for all orderings in I and for all p in P,

and that

$(a, r) \succsim (a, q) \succsim (a, p)$ for all orderings in I and all a in A.

Now consider the following pair comparisons from three different orderings:

$$(a, q) >_i (b, p) \text{ and } (c, p) \succsim_i (b, r);$$
$$(a, r) >_j (c, p) \text{ and } (b, q) \succsim_j (a, r);$$
$$(b, r) >_k (c, q) \text{ and } (c, p) >_k (a, q).$$

None of these pairs violates any of the axioms of additive conjoint measurement implied by Axiom 1. Also, it can be verified that there are no violations of Axiom 3. Yet, as will be shown below, these three orderings cannot be accommodated simultaneously within the context of a weighted linear model.

If the weighted linear model is assumed to apply, then $(a, q) >_i (b, p)$ implies that there exists a w such that

$$wf(a) + (1-w)g(q) > wf(b) + (1-w)g(p).$$

Simple algebra reduces this to

$$w[f(b) - f(a) + g(q) - g(p)] < g(q) - g(p).$$

The term inside the brackets on the left must be greater than zero because of the above assumptions about the ordering of attribute levels. Thus,

$$w < [g(q) - g(p)] / [f(b) - f(a) + g(q) - g(p)].$$

To simplify notation, let $x = f(b) - f(a)$, $y = f(c) - f(b)$, $u = g(q) - g(p)$, and $v = g(r) - g(q)$ so that $x + y = f(c) - f(a)$ and $u + v = g(r) - g(p)$. Because of the assumptions about the attribute orderings, $x, y, u, v > 0$. Thus,

$$w < u/(x + u).$$

Similarly, $(c, p) \succsim_i (b, r)$ for ordering i implies that for the <u>same</u> w

$$wf(c) + (1-w)g(p) \geq wf(b) + (1-w)g(r)$$

which, using the above notation, reduces to

$$w \geq (u + v)/(y + u + v).$$

Because w must be the same in the last two inequalitites, it follows that

$$(u + v)/(y + u + v) < u/(x + u) \text{ or}$$

$$x(u + v) < yu.$$

A similar exercise for the two pair comparisons from ordering j yields the inequality

$$yv < xu.$$

Note that these last two inequalities involve only the parameters x, y, u, and v, which correspond to differences between scale values, and that they do not depend upon the particular weight associated with the ordering. Later we will show how such inequalities can be solved to yield a numerical scaling. Because these inequalities do not depend upon the weight they can be added to obtain, after appropriate cancellations,

$$v(x + y) < yu.$$

Finally, we can apply the same procedures to the two pair comparisons from ordering k. This yields

$$yu < v(x + y)$$

which directly contradicts the previous inequality. Thus, the three orderings i, j, and k, although consistent with Axioms 1, 2, and 3, are not compatible as a group with the weighted linear model. The following axiom excludes this particular incompatibility.

<u>Axiom 4</u>: <u>Three-ordering compatibility</u>.

For all a, b, c in A; and all p, q, r in P:

i) if there exists an ordering i in I such that

$(a, q) >_i (b, p)$ and $(c, p) \not>_i (b, r)$

and if there exists an ordering j in I such that

$(a, r) >_j (c, p)$ and $(b, q) \not>_j (a, r)$

then for all orderings k in I

$(b, r) >_k (c, q)$ implies $(a, q) >_k (c, p)$.

Axiom 4(i) excludes the incompatibility demonstrated above. Unfortunately, there are seven other similar incompatibilities and there does not appear to be a general expression that can be applied to all; thus, Axiom 4 has seven other parts. We state these additional parts in abbreviated form; the full statement would parallel that in Axiom 4(i).

<u>Axiom 4</u>: (Continued)

ii) (a, r) > (b, q) and (c, p) $\gtrsim$ (b, r) for ordering i, and (a, r) > (c, p) and (b, p) $\gtrsim$ (a, q) for ordering j, then (b, q) > (c, p) implies (a, r) > (c, q) for ordering k.

iii) (a, r) > (b, p) and (c, p) $\gtrsim$ (a, r) for ordering i, and (b, r) > (c, p) and (b, p) $\gtrsim$ (a, q) for ordering j, then (a, q) > (c, p) implies (b, r) > (c, q) for ordering k.

iv) (b, r) > (c, p) and (c, p) $\gtrsim$ (a, r) for ordering i, and (a, q) > (b, p) and (c, p) $\gtrsim$ (a, r) for ordering j, then (a, r) > (c, q) implies (b, q) > (c, p) for ordering k.

v) (b, r) > (c, q) and (b, p) $\gtrsim$ (a, r) for ordering i, and (a, r) > (c, p) and (c, p) $\gtrsim$ (b, q) for ordering j, then (a, q) > (b, p) implies (a, r) > (c, q) for ordering k.

vi) (b, q) > (c, p) and (b, p) $\gtrsim$ (a, r) for ordering i, and (a, r) > (c, p) and (c, p) $\gtrsim$ (b, r) for ordering j, then (a, r) > (b, q) implies (a, q) > (c, p) for ordering k.

vii) (b, q) > (c, p) and (c, p) $\gtrsim$ (a, r) for ordering i, and (a, r) > (b, p) and (c, q) $\gtrsim$ (b, r) for ordering j, then (a, r) > (c, q) implies (a, q) > (b, p) for ordering k.

viii) (a, r) > (b, p) and (c, p) $\gtrsim$ (b, q) for ordering i, and (b, r) > (c, q) and (c, p) $\gtrsim$ (a, r) for ordering j, then (a, q) > (c, p) implies (a, r) > (b, q) for ordering k.

The Theorems

The major results can be stated in terms of three theorems.

Theorem 1: Necessary Conditions. If <<A x P, I>> is a weighted linear structure, then Axioms 1-4 must be satisified.

Theorem 2: Sufficient Conditions. If A and P both contain exactly three elements, then Axioms 1-4 are both necessary and sufficient to establish that <<A x P, I>> is a weighted linear structure.

Theorem 3: Uniqueness. If real-valued functions f and g and parameters $W = \{w_i, w_j, \ldots\}$ provide a numerical representation for a weighted linear structure, then the following transformations provide another:

$$f' = \alpha_1 f + \beta_1, \quad \alpha_1 > 0$$

$$g' = \alpha_2 g + \beta_2, \quad \alpha_2 > 0$$

$$w_i' = \left[1 + \left(\frac{\alpha_1}{\alpha_2}\right)\frac{1 - w_i}{w_i}\right]^{-1}.$$

That is, the scale values given by f and g are unique up to a positive linear transformation and the weights are unique only up to a nonlinear, but monotonic, transformation.

Proofs: Theorem 1 was essentially proved as the axioms were introduced above. Theorem 3 involves substituting the specified transformation into the weighted linear model as defined above and then reducing the resulting structure to the original. This simple algebraic proof is omitted. The proof of Theorem 2 is an inelegant, brute-force procedure for considering all possible cases. While we have done all the constructions described below, we present the proof in outline only.

<u>Proof of Theorem 2</u>. Axioms 1 and 2 ensure that it is meaningful to talk about intersections and that, in particular, for a complete set of orderings (e.g., one ordering from each interval in Figure 1) there are exactly nine intersections. Each ordering of the intersections is a potential data structure (i.e., each might correspond to a complete representation). Axiom 3 guarantees that the ordering of the intersections is a weak order. However, there are 9! = 362,880 possible weak orderings of intersections, many of which involve incompatibilities. An examination of each possible ordering (using backtrack programming techniques such as those used in McClelland, 1977, to count additive orderings) demonstrates that the application of Axiom 3 reduces the number of compatible sets of orderings from 9! to 48. This again shows the importance of Axiom 3 and justifies its label as the "weighting axiom". Application of Axiom 4 eliminates exactly eight orderings (corresponding to the eight parts of Axiom 4), leaving only 40 intersection orderings. These 40 orderings each generate a system of nonlinear inequalities (like obtained in the proof for Axiom 4), all of which have solutions. Hence, there are exactly 40 complete sets of alternative orderings, each corresponding to an ordering of intersections, that are consistent with the weighted linear model. Thus, Axioms 1-4 are sufficient for the 3 x 3 case of the weighted linear model.

Theorem 2 for the 3 x 3 case and its proof are not very interesting in themselves other than to show that this relatively small set of axioms does cover all the possibilities for the 3 x 3 case. For comparison, Axioms 1-4 for the weighted linear model are comparable to the monotonicity and double cancellation axioms for additive conjoint measurement in that they are both necessary and sufficient for the 3 x 3 case. The most interesting aspect of the proof of Theorem 2 is the construction of numerical representations for the 40 consistent sets of orderings. Below we illustrate how this scaling can be accomplished using the record-book orderings from Table 2.

Scaling Procedures

Application of Axioms 1-4 to the orderings of alternatives in Table 2 ensures that it is possible to estimate the weights and scale values associated with the orderings. The ordering of pair reversals corresponds to the ordering of intersections which is listed in the second column of Table 3. Because Theorem 3 indicates that the scale values will be unique only up to a linear transformation, we will scale the differences between attribute levels and fix the units of each scale in the following manner: Let $f(c) - f(a) = x + y = 1$ and $g(r) - g(p) = u + v = 1$, where x, y, u, and v are as defined above, so that $y = 1 - x$ and $v = 1 - u$.

Next, solve for the value of the weight parameter at each intersection. For example, at the first intersection (CD),

$$(c, p) \approx (b, r) \text{ or}$$

$$wf(c) + (1-w)g(p) = wf(b) + (1-w)g(r)$$

which with the above substitution reduces to

$$w = 1/(2-x).$$

The third column of Table 3 lists the corresponding expression for the weight for each intersection.

The ordering of the intersections also orders the weights. That is,

$$1 > w_1 > w_2 > \ldots > w_8 > w_9 > 0.$$

Substituting the expressions for the weights yields a system of nonlinear inequalities. For example, $w_1 > w_2$ implies that $1/(2-x) > 1/(1+x)$ or equivalently, $x > .5$. For the ordering of weights in Table 3 this system reduces to the following set of nonredundant inequalities:

$$1-x > x(1-u)$$

$$u < 1/2$$

$$x > 1-u.$$

Many standard programming techniques are available for finding solutions to such systems of nonlinear inequalities. Figure 3 presents a graphic solution. The solution space (i.e., the possible values for x and u) is the shaded area bounded by the three inequalities. Any point within the solution space represents a numerical scaling consistent with the orderings of Table 2. For example, the point $x = .6$, $u = .45$ is in the solution space. In terms of the evaluation function this means that the distances between attribute levels are as follows: 0 to 2 records = .6, 2 to 5 records = .4, 0 to 1 books = .45, and 1 to 5 books = .55. These are in fact the values that were used to construct Figure 1.

The weights which define the boundaries between orderings are expressed in Table 3 in terms of x and u. Substituting $x = .6$ and $u = .45$ provides the numerical estimates of the boundary weights listed in the fourth column of Table 3. In terms of Figure 1, these weights represent the distance of the intersection from the line representing zero weight on records.

These scale values can also be used to characterize the individual orderings. For example, consider ordering #4 in Table 2. Because CD, FG, and BD have been reversed, this ordering must be in the slice between the third and fourth intersections. Thus, the weight associated with this ordering is between the corresponding weights for those two intersections, .53 and .58. (Note that for all but the first and last slices the range for the weight is fairly small.) Let's choose .55, the midpoint of the range, to represent the fourth ordering. To fix the origins of the two scales let $f(a) = g(p) = 0$. Then the numerical evaluation can be calculated for each alternative using the weighted linear model. In this case, $A = 1$, $B = .75$, $C = .55$, $D = .78$, $E = .53$, $F = .33$, $G = .45$, $H = .20$, and $I = 0$. This set of scale values does indeed reproduce the ordering of the alternatives: ADBCEGFHI. Scale values for the other orderings can be computed similarly.

Discussion

Our discussion is in two parts: (a) a comparison of the present axiomatization with Luce's (1980) axiomatization of the averaging model, and (b) a consideration of implications of this axiomatization for some of the important disagreements among various approaches to judgment and decision making.

Comparison to Luce (1980). Luce provides a set of axioms for the averaging and adding representations of functional measurement. Whereas our axioms apply only in the case of exactly two attributes, Luce allows the number of attributes to vary. That is, an individual might be asked to compare two alternatives described by different numbers of attributes. In our axioms, an individual is always presumed to be comparing two alternatives described by the same two attributes. Both types of tasks have been used in judgment and decision making studies so it is useful to have an axiomatization for each.

Another way in which the two axiomatizations differ is that Luce considers the infinite case while our results (particularly Theorem 2) pertain mostly to small, finite cases. As noted above, there are advantages and disadvantages to each approach. The advantages and disadvantages are clearly illustrated by the two axiomatizations. The infinite case often leads to a better theoretical understanding of the measurement structure involved. In this instance, Luce was able to show that with appropriate definitions the averaging model is a special case of the conditional expected utility model presented in Krantz, Luce, Suppes, and Tversky (1971). By showing the strong relationship between the weighted linear model and another measurement structure, a better understanding is immediately obtained, and many results from the other measurement structure can be applied. Such strong results are sometimes, but not often, obtained with the finite case. We did suggest that there was an analogy between our axioms and unfolding theory, but the link is much weaker than the link obtained by Luce.

On the other hand, an important disadvantage of considering the infinite case is that the resulting axioms are often not testable empirically. Luce uses the standard Archimedean axiom and a restricted solvability axiom which are not directly testable; rather, one decides on the basis of the experimental context whether they are appropriate (this issue is discussed in Krantz, et al., 1971, and Krantz & Tversky, 1971). Further, Luce's key axiom, which distinguishes adding from averaging, is defined in terms of equivalences. A direct test of this axiom would be very difficult experimentally because it requires finding two alternatives, defined in terms of distinct sets of attributes, which the decision maker judges to be equivalent. For the averaging model to hold, the decision maker must judge the union of the two alternatives to be equivalent to either single alternative. Even in laboratory studies of judgment and decision making, finding such equivalences would be extremely difficult. In contrast, the axioms presented here are easily and directly testable; several such tests were illustrated above. However, as is true with all finite axiomatizations, our axioms are undoubtedly not sufficient for cases with either more than two attributes or more than three attribute levels. The necessary conditions would of course still apply to larger cases.

Despite the differences in the two axiomatizations, the implications and conclusions from both are virtually identical. Most importantly, both demonstrate that the adding and averaging models are distinguishable (with only ordinal data) and both show that with appropriate operationalizations and with enough data, weights can be identifiable parameters. That is, weights need not be mere scaling constants. We discuss this issue in greater detail below.

Implications. It is hoped that the axiomatization presented here will aid in identifying and exploring common ground and disagreements among the various approaches to judgment and decision making. Not all such implications of the results can be considered in a paper of this scope. Instead, we present as examples three implications of the axiomatization of the weighted linear model. These implications pertain to (a) the identifiability of weights, (b) the uniqueness of estimates of weights and scale values, and (c) the effect of the range of attribute levels on weight.

1. The identifiability of weights. Our axiomatization provides clear evidence that (in the context of the weighted linear model) weights can be more than mere "scaling constants" and that they need not be "empirically empty parameters". The results show that weight is an identifiable parameter--numerical estimates of the value of a weight can be derived from data.

Of course, just because weight can be an identifiable parameter does not mean that weight will be meaningful in all the diverse operationalizations of the weighted linear model. Indeed, our results and those of Luce (1980) strongly suggest that meaningful weights cannot be obtained by one data collection method commonly used in judgment and decision making research--presenting alternatives defined by a fixed set of attributes to one person in one situation. Thus, Keeney and Raiffa (1976) are correct when they emphasize that the weight parameters obtained through this method are indeed no more than scaling constants. However, our results suggest that the weight parameters could be more than scaling constants if additional information were collected. For example, weights could be estimated if the decision maker made additional choices under different conditions. Or it might be possible to construct a representation for several different decision makers for the same decision problem.

The ability to estimate meaningful weights is achieved at a price, however, in this axiomatization. The price is that the same scale values or evaluation functions must be used for all orderings whether those orderings are from one or many people. There are clearly many judgment and decision problems for which it would be unreasonable to expect that the same scale values would apply to all orderings. Indeed, several studies using the approach of social judgment theory (e.g., Balke, Hammond, & Meyer, 1973; Rohrbaugh & Wehr, 1978.) have shown that policy makers sometimes have very different evaluation functions for individual attributes; in some cases they do not even agree on the monotonic ordering of the attribute levels (in violation of Axiom 2). For example, some urban policy makers always prefer more growth while others prefer moderate amounts of growth to either extreme--a clear violation of monotonicity. However, it is not apparent (at least not from this axiomatization) whether it is meaningful to compare weights when the attribute evaluation functions are very different.

On the other hand, there are judgment and decision problems for which it would be reasonable to expect common scale values. This expectation would certainly be reasonable if all the orderings were produced by the same person under different conditions. For example, a person asked to evaluate a number of automobiles under each of a number of assumptions about future supplies of gasoline might well use the same scale values (for the levels of such attributes as comfort, fuel efficiency, and safety) for each set of evaluations, with only the weight accorded each attribute varying with the fuel supply scenario. Several information integration studies (e.g., Anderson, 1964; Shanteau, 1972; Weiss & Anderson, 1969) have shown implicitly that scale values are constant across one type of change in the situation in which judgments are collected--a change in the serial position of the attibutes. Also, there are judgment and decision problems for which the assumption that people differ only in their weights is not unreasonable. To reiterate, even when common scale values can be assumed, our axiomatization shows only that weights _might_ be estimable; whether or not they actually can be estimated is an empirical question to be decided by testing the axioms.

2. _Uniqueness of weights and scale values._ The uniqueness

theorem (Theorem 3) has some surprising implications. In additive conjoint measurement, the functions for each attribute are unique only up to a positive linear transformation; this is also the case for the weighted linear model presented here. However, in additive conjoint measurement the two functions have the same unit of measurement (i.e., if one scale is multiplied by a constant then the other must be multiplied by the same constant). For the weighted linear model the two functions need not have the same unit of measurement--each could be transformed by multiplying by different positive constants. The reason is that an appropriate transformation of the weight (described in Theorem 3 in terms of the two multiplicative constants) can always make the scales commensurate again. That is, of course, precisely the role that the weights are supposed to play both mathematically and psychologically. However, this also means that the weights are unique only up to a nonlinear, but monotonic, transformation. A major implication of this characteristic is that certain types of statements about "relative weights" cannot be meaningful. For example, according to the uniqueness theorem, it makes no sense to say that one person places a greater weight on attribute A than on P. The problems involved with such relative comparisons are best illustrated by returning to the record-book example.

Theorem 3 says the evaluation functions are unique only up to a positive linear transformation. If the evaluation function for the book attribute is transformed so that $f(c) - f(a) = 10$ instead of 1 (i.e., all scale values are multiplied by 10) then the nine boundary weights listed in Table 3 become .985, .945, .936, .923, .916, .911, .892, .855, .840. Thus, all orderings (except possibly the tenth) appear to place greater weight on records than on books because all the boundary weights are greater than .5. It is thus clear that the weights are dependent upon the particular interval scale chosen for the evaluation functions. Hence, statements about relative weights across attributes are not meaningful because they can always be reversed by an appropriate choice of a permissible linear transformation. Keeney and Raiffa (1976, pp. 271-273) provide a similar example of the impossibility of such relative comparisons.

Meaningful statements about relative weights across orderings are possible. For example, if ordering S1 has fewer pair reversals (relative to a lexicographic ordering) than S2, it is meaningful to say that S1 places more weight on attribute A than does S2.

The implications of the uniqueness theorem are surprising because the restrictions on the types of relative comparisons that are possible are exactly opposite to what is generally assumed. Relative comparisons of weights across attributes are commonly reported, while relative comparisons across situations or people are less frequent.

3. Range effects. Much recent debate has centered on whether changes in the ranges of attribute levels can, should, or must affect estimates of weight (e.g., Levin, Kim, & Corry, 1976; Gabrielli & von Winterfeldt, 1978). While the issue may be in doubt for some operationalizations of the weighted linear model, the answer is clear for our axiomatization. Consider the representation of a weighted

linear model in Figure 1. If a fourth attribute level were added to each attribute above the current maximums (say, eight records and ten books), then it is obvious that the original nine intersections would be unchanged. In particular, the intersections would still occur in the same order. Of course, there would now be additional intersections inserted into the order; these new intersections would allow finer discriminations because there would be 17 slices instead of ten. However, this is not cause for the decision maker to behave diferently toward the original nine alternatives.

To put it another way, if the weights are to be estimated from a given 3 x 3 subset of the attribute levels, only the orderings within that subset will affect the weights. Thus, as long as the orderings within the subset remain the same it makes no difference in what larger set the subset is included--the weights will remain the same. Of course, it is an empirical question as to whether exposing a decision maker to a larger range will cause a change in weights (e.g., as might result from including death as one extreme for an attribute). The axiomatization, in fact, suggests how to do this experiment: embed a given 3 x 3 subset in larger sets which vary the range of the attribute levels. If the orderings within the 3 x 3 subset do not vary across sets, then there has not been a true change in weights. If the evaluation functions change with attribute-level range, the orderings will violate the axioms. If the orderings within the subset change but remain consistent with the axiomatization, then the scaling procedures outlined above can be used to track the changes in weight as a function of the changes in range.

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Table 1

Alternatives and attribute levels for the record-book example

Alternative	Records	Books
A = (c, r)	5	5
B = (c, q)	5	1
C = (c, p)	5	0
D = (b, r)	2	5
E = (b, q)	2	1
F = (b, p)	2	0
G = (a, r)	0	5
H = (a, q)	0	1
I = (a, p)	0	0

Table 2

Orderings of alternatives defined by the ordering of intersections depicted in Figure 1 (The new pair reversal in each ordering is underlined)

No.	Ordering	
1	A B C D E F G H I	(high weight on records)
2	A B D C E F G H I	
3	A B D C E G F H I	
4	A D B C E G F H I	
5	A D B E C G F H I	
6	A D B E G C F H I	
7	A D B G E C F H I	
8	A D B G E C H F I	
9	A D G B E C H F I	
10	A D G B E H C F I	(high weight on books)

Table 3

Ordering of intersections and the corresponding weights derived from the alternative orderings of Table 2

No.	Intersection	Boundary Weight	Weight Estimate
1	CD	1/(2-x)	.71
2	FG	1/(1+x)	.63
3	BD	(1-u)/(2-x-u)	.58
4	CE	u/(1-x+u)	.53
5	CG	1/2	.50
6	EG	(1-u)/(1+x-u)	.48
7	FH	u/(x + u)	.43
8	BG	(1-u)/(2-u)	.35
9	CH	u/(1+u)	.31

Figure 1

A hypothetical scaling for the record-book example

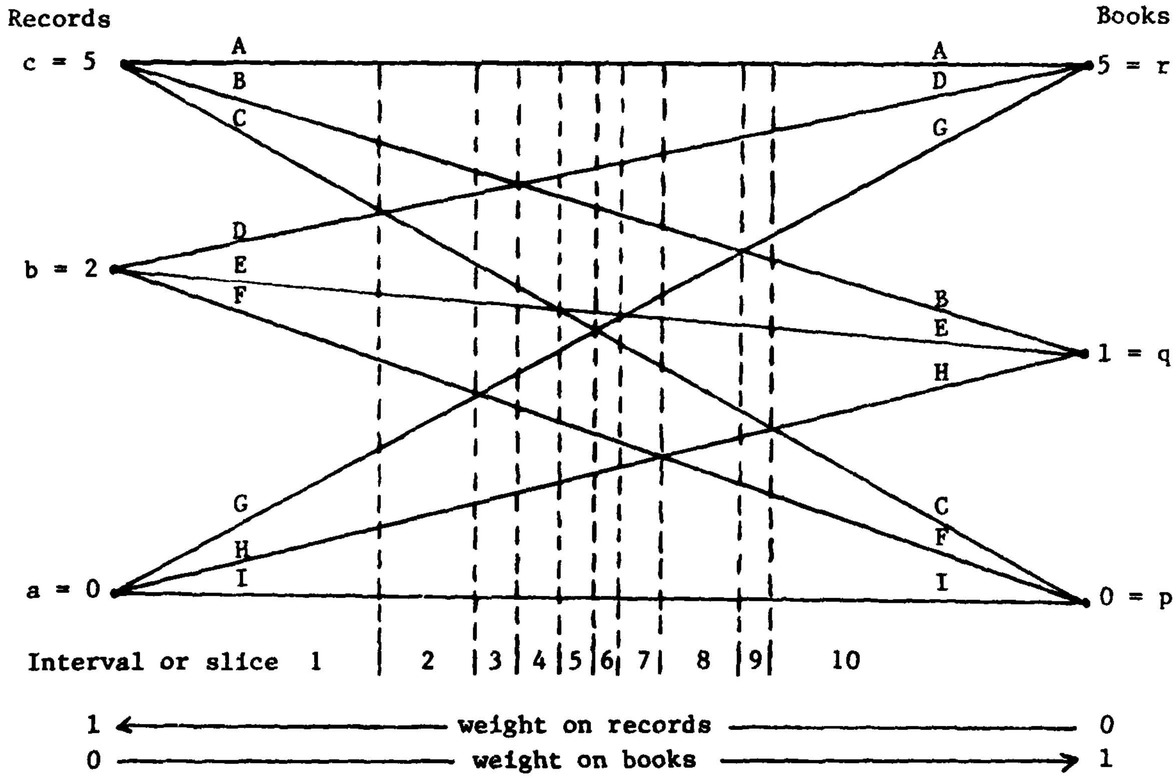
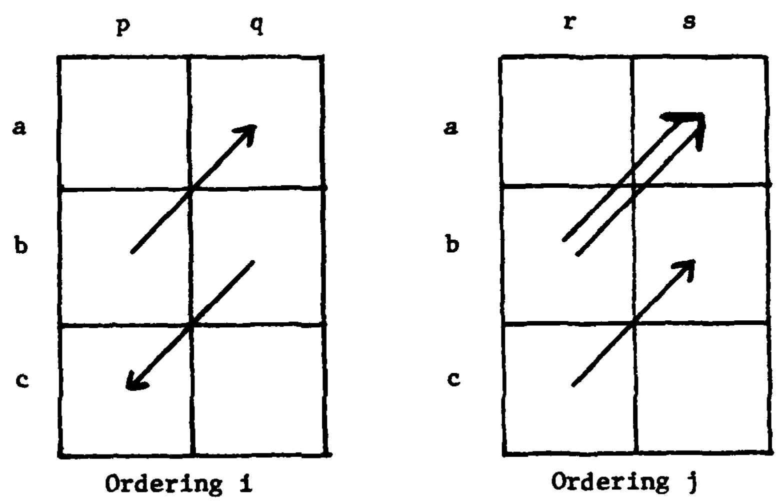


Figure 2

Graphic representations of Axiom 3:

Cross-ordering compatibility

Axiom 3(i)



Axiom 3(ii)

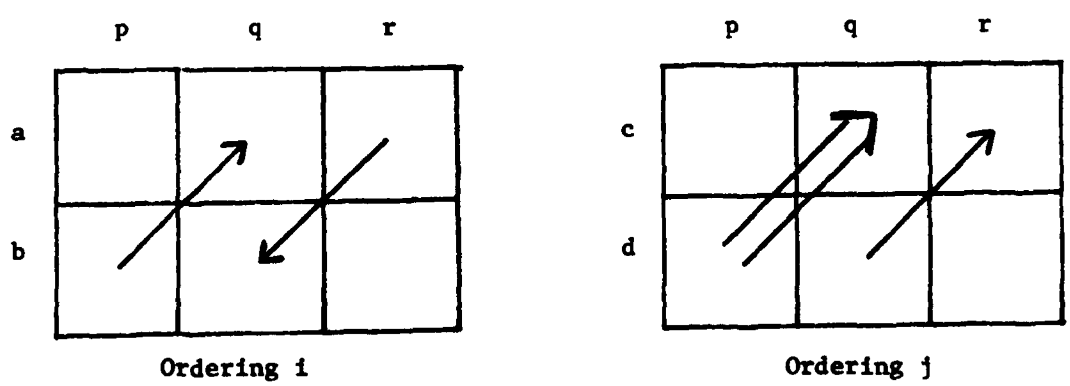
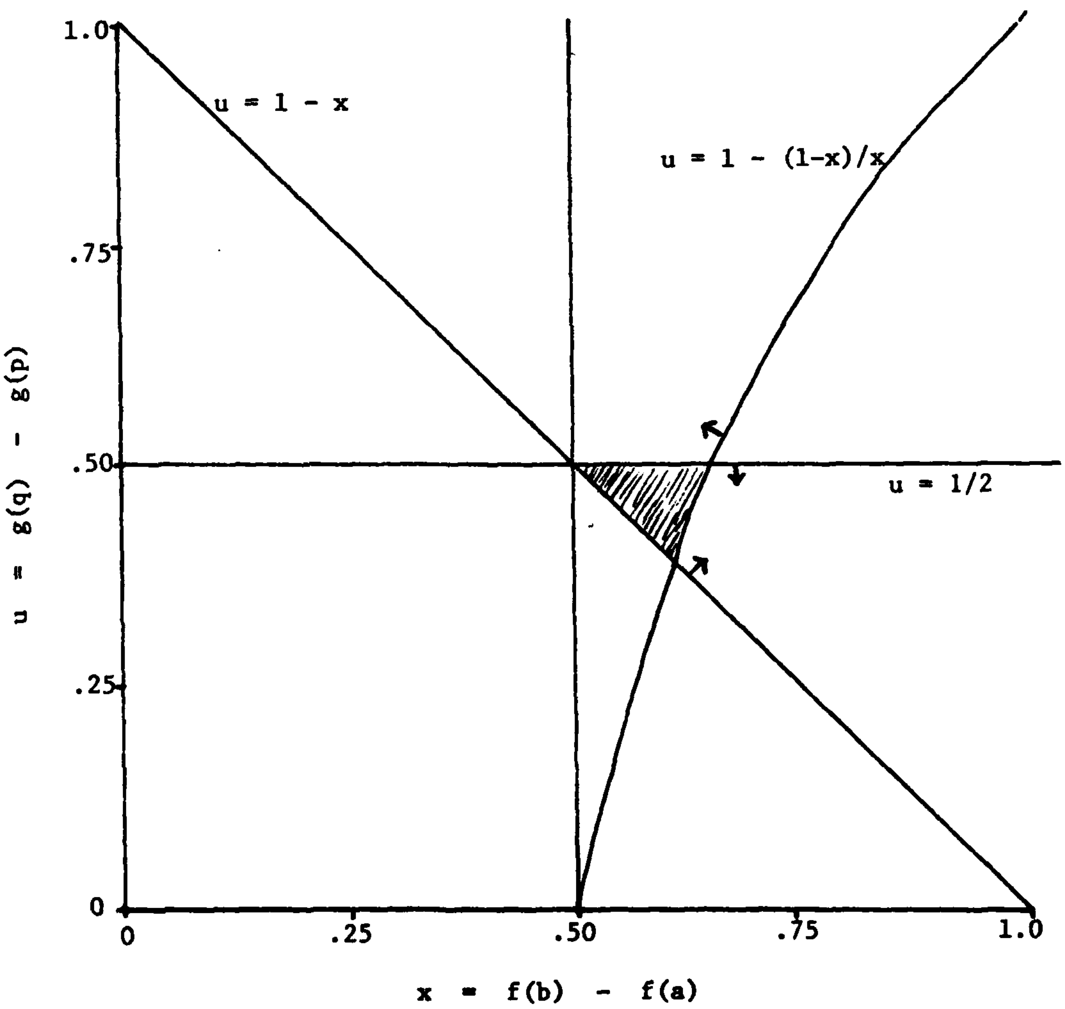


Figure 3

Solution space implied by the alternative orderings of Table 2



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